Quantized spin wave modes in magnetic tunnel junction nanopillars

A. Helmer,1,2, ∗ S. Cornelissen,3,4 T. Devolder,1,2 J.-V. Kim,1,2 W. van Roy,3 L. Lagae,3,5 and C. Chappert1,2

1 Institut d’Electronique Fondamentale, UMR CNRS 8622, 91405 Orsay, France
2 Université Paris-Sud, 91405 Orsay, France
3 IMEC, FNS, Kapeldreef 75, 3001 Leuven, Belgium
4 ESAT, KU Leuven, Leuven, Belgium
5 Natuurkunde en Sterrenkunde, KU Leuven, Leuven, Belgium

(Dated: July 10, 2009)

We present an experimental and theoretical study of the magnetic field dependence of the mode frequency of thermally excited spin waves in rectangular shaped nanopillars of lateral sizes 60 × 100, 75 × 150, and 105 × 190 nm², patterned from MgO-based magnetic tunnel junctions. The spin wave frequencies were measured using spectrally resolved electrical noise measurements. In all spectra, several independent quantized spin wave modes have been observed and could be identified as eigenexcitations of the free layer and of the synthetic antiferromagnet of the junction. Using a theoretical approach based on the diagonalization of the dynamical matrix of a system of three coupled, spatially confined magnetic layers, we have modeled the spectra for the smallest pillar and have extracted its material parameters. The magnetization and exchange stiffness constant of the CoFeB free layer are thereby found to be substantially reduced compared to the corresponding thin film values. Moreover, we could infer that the pinning of the magnetization at the lateral boundaries must be weak and slightly asymmetric. At low fields and in the larger pillars, there is clear evidence for strong nonuniformities of the layer magnetizations. In particular, at zero field the lowest mode is in many devices not the fundamental mode, but a mode most likely localized near the layer edges.

PACS numbers: 75.75.+a, 75.60.-d, 72.25.Pn, 85.75.-d

I. INTRODUCTION

In the last few years, magnetic tunnel junction (MTJ) nanopillars have received tremendous attention due to their promising potential for applications in spin-transfer-switched Magnetic Random Access Memory or as spin-torque oscillators for microwave generation.1–4 With GHz frequencies the operation speed of these devices happens to lie in the same frequency range as the dynamic eigenexcitations of the underlying nanoelements (thermally excited spin waves), which may therefore manifest themselves as unwanted noise sources. However, as eigenexcitations, thermal spin waves also constitute an excellent probe for the intrinsic magnetic properties of the nanopillars, which are not accessible otherwise. The experimental detection of spin waves in MTJ nanopillar devices and the understanding of their nature is therefore of great interest both for fundamental and technological reasons.

Spin waves in confined structures have been studied extensively in single-layer dots with thicknesses between 40 and 15 nm and typical lateral dimensions from 3 μm down to 200 nm.5–8 In these systems, two types of spin wave modes have been identified: quantized volume modes located around the center of the element where the internal field is basically homogeneous, and spin wave well or edge modes localized near the element edges in the inhomogeneity region of the internal field. The above elements are characterized by their thickness being significantly larger than the exchange length of the layer material (NiFe) being about 5 nm. In structures with this property the dominating interaction is the magneto-static dipolar interaction,9 which causes the inhomogeneity of the internal field, thus determining the character and spatial profile of the modes.4

The eigenexcitations of a multi-layer dot differ in general significantly from those of an ensemble of isolated magnetic dots due to the interlayer interactions between the magnetic layers in the stack: mutual dipolar coupling and - for sufficiently thin metallic spacer layers - interlayer exchange coupling.10 Eigenexcitations of nanopillar structures have been the subject of very few studies so far. Thermal spin waves have been investigated systematically only in pseudo-spin-valves11–13 of circular and elliptical shape (smallest dimension 200 nm) consisting of two magnetic layers of 10 nm thickness separated by a 10 nm thick spacer layer, i.e. again layer thicknesses were much larger than the exchange length. Consequently, the profiles of the modes in each of the two pillar layers showed great resemblance7,12 with the mode profiles in the corresponding isolated dots. In a symmetric spin-valve stack,11 the main impact of the mutual dipolar coupling between the layers was found to be a fixed phase relation between the modes in the two layers for high applied field, and hybridization effects at low field. Finally, the spin wave modes in an asymmetric spin-valve stack13 turned out to be very different from those in the corresponding symmetric spin-valve.11

Common MTJ nanopillars differ qualitatively from the pseudo-spin-valves in three fundamental points: Firstly, with a free layer and a synthetic antiferromagnet (SAF) they consist of three magnetic layers; secondly, with 2 – 4 nm the layer thicknesses are smaller than the exchange length,14 such that, for sufficiently small lateral dimensions, the spin dynamics in each layer is now dominated by the exchange...
interaction; thirdly, the interlayer interaction of the layers is highly asymmetric, as the two SAF layers are coupled by both interlayer exchange and mutual dipolar coupling, one layer being additionally subject to the strong exchange bias field, whereas the free layer interacts with the other two layers via mutual dipolar coupling only. Finally, the pillar stack itself is in general totally asymmetric, all three layers being different from each other. The eigenexcitations of an MTJ nanopillar are therefore expected to be much more complex than those of pseudo-spin-valves.

In this paper, we investigate the magnetic field dependence of the mode frequency of thermally excited spin waves in rectangular shaped MgO-based MTJ nanopillars of different lateral sizes. In section II, we will describe the basic magnetic properties of the devices and the experimental techniques used to acquire the spin wave spectra. The features of the measured spectra in dependence of the pillar size and the direction of the applied field are described in the following section III. In section IV, we will point out short-comes of the macrospin model when applied to our samples, as a consequence of which we will introduce in section V a model of quantized spin wave modes in nanopillars consisting of three magnetic layers. In section VI, we will use this model to extract the material parameters of the pillar, which will finally be discussed in section VII along with the limitations of our model.

II. SAMPLES AND EXPERIMENTAL TECHNIQUES

A. Samples and basic device properties

The fabrication and basic properties of our samples are described in Ref. 15: they are rectangular shaped nanopillars, all patterned from the same MTJ stack of composition $\text{Co}_{20}\text{Fe}_{20}\text{B}_{20}$ (3 nm, free layer)/ Mg(1.3)[nat. ox.]/ $\text{Co}_{20}\text{Fe}_{20}\text{B}_{20}$ (2 nm, reference layer)/ Ru(0.8)/ $\text{Co}_{20}\text{Fe}_{20}$ (2 nm, pinned layer)/ PtMn(20), deposited by Singulus Technologies AG. The three layers following the MgO tunnel barrier compose the synthetic antiferromagnet (SAF). The pillars were designed in three lateral sizes: 60 × 100, 75 × 150, and 105 × 190 nm², which will be referred to as small (S), medium (M), and large (L) size, respectively. Note that unlike in Ref. 15 the given dimensions are not the nominal values, but mean values measured on the exposed e-beam medium (M), and large (L) size, respectively. Note that unlike in Ref. 15 the given dimensions are not the nominal values, but mean values measured on the exposed e-beam resist with a device-to-device deviation of ±10 nm. In order to obtain electrically contactable devices the nanopillars were inserted in series between coplanar leads, following design rules ensuring high bandwidth.16

The devices have a resistance area product of 16 $\Omega \mu\text{m}^2$ and typically 80 – 120% tunnelling magneto-resistance ratio. Their hysteretic properties are consistent with the uniaxial anisotropy expected from the rectangular pillar shape where the long edge of the rectangle, oriented along the exchange pinning direction of the PtMn antiferromagnet, is the easy axis (EA), and the short edge the hard axis (HA) of the magnetization. Panels (b) and (d) of Fig. 1 show as a reference EA and HA hysteresis loops of a rectangular nanopillar calculated in macrospin approximation; in comparison, the corresponding experimental loops are depicted in Figs. 2-4.

Fig. 2(c) shows the EA hysteresis loop of a device of pillar size S, and Figs. 3(b) and (d) the ascending field branches of the EA loops for pillar sizes M and L, respectively. At negative fields the devices are in the parallel (P) state, at positive fields in the antiparallel (AP) state. Spin-flop (SF) transition of the SAF occurs typically at EA fields around +140 mT. Room temperature coercivity is 25 – 35 mT for devices of size S and M, and 20 – 25 mT for size L. From astroid measurements15 mean anisotropy fields of 37 mT, 46 mT, and 38 mT for pillar sizes S, M, and L, respectively, have been determined. The EA loops for devices of size S are off-centered towards negative fields by 5 – 11 mT indicating nonnegligible antiparallel coupling of the free layer magnetization and the SAF. With increasing pillar size this coupling is decreasing: for size M the shift is only 3 – 7 mT, and for size L 0 – 3 mT. The bell shape of the HA hysteresis loops (figs. 4b,d,f) is in agreement with the antiparallel coupling observed on the EA: For zero applied field the devices are in the high resistance AP state, while with increasing (absolute value of the) field the resistance decreases continuously almost down to parallel remanence, as the magnetizations of the free layer and finally the two SAF layers progressively tilt towards the applied field. Note that to reach the P configuration on the HA, much higher fields are needed than those available on our set-up (±160 mT). Ascending and descending field branch of the HA loops are in general identical.

We have used the intrinsic symmetry of the HA loops to align the external field with the symmetry axes of the rectangle by choosing the field direction such that the loops showed highest possible symmetry, in particular at high fields. The misalignment therefore should not exceed $3^\circ$.

B. Set-up and experimental methods

To obtain their spin wave spectra the devices were inserted into a high bandwidth circuit similar to that in Ref. 17, and their voltage noise power spectrum density (PSD) was measured for moderate dc bias currents as a function of the applied magnetic field. The noise spectrum at each field step was obtained by subtracting from an averaged spectrum for non-zero bias current a subsequently measured averaged zero-current reference spectrum. Therefore, the spectra are displayed using a contrast scaling with the logarithm of the noise in excess to the noise for zero current. Bias currents were chosen as low as possible in order not to affect the mode frequencies, but still high enough to obtain sufficient signal-to-noise ratio. Devices of pillar sizes S and M were therefore mostly measured at ±0.1 mA, those of pillar size L usually at ±0.3 mA: For 0.1 mA, no measurable differences between spectra of opposite current polarity have been observed. For 0.3 mA, in some devices, the mode frequency undergoes a minimal shift when the current polarity is inverted, while in other devices it remains unchanged. In neither case, any noticeable difference in the mode intensity
could be detected. This finding is in agreement with previous works\textsuperscript{19} on similar samples, where an spin-torque threshold current of 1.6 mA for size-L devices has been determined. The bias current dependence of the spectra can therefore be neglected in the scope of this paper.

In figs. 2(a),(b), 3(a),(c) and 4(a),(c),(e) examples of 2D density plots of the PSD versus the magnetic field are shown, where the dark regions correspond to maxima in the PSD and therefore to eigenexcitations of the magnetic system\textsuperscript{19} i.e. spin wave modes. In all figures black dots have been superimposed to better evidence the modes.

Note that our measurement technique allows to detect the spin wave spectra of individual pillars, in contrast to the Brillouin Light Scattering technique used in Refs. 4,11,13, or the frequency-domain coplanar waveguide technique of Ref. 5, where the measured spectra were an average over a large number of devices. Moreover, we do not need optical or any other direct access to the magnetic layers of the pillar, but can measure them in their natural working environment, i.e. as part of the actually used stack and biased with an electrical current. Finally, since we measure the voltage noise, which, for small bias currents, is proportional to the magneto-resistance (MR) noise, we are sensitive to spin waves both in the free layer and the reference layer.

### III. EXPERIMENTAL RESULTS

In this section, we describe the characteristics of the spectra measured for easy axis and hard axis applied fields as well as their dependence on the pillar size. For the basic identification of the observed spin wave modes we recall in Fig. 1 the mode dispersion of a nanopillar in macrospin approximation.

#### A. Eigenexcitations for easy axis applied field

1. **Size independent properties**

For all pillar sizes, the easy axis spectra, Figs. 2 and 3, contain basically two groups of modes. The first group consists of V-shaped modes essentially symmetric about zero-field for high positive and negative fields and history dependent in the hysteretic field region with a discontinuity at the coercivity field (compare panels (a),(b), and (c) of Fig. 2). Since this is the behavior expected from free layer (FL) modes (Fig. 1(a),(b)), the modes in this group will be labeled with F, where F0 is the lowest (and most intense) FL mode. The second group consists of modes having a minimum at the spin-flop field. This being the expectation for acoustic modes of the SAF (Fig. 1(a),(b)), the modes in this group will be labeled with A. The spacing between both FL and SAF modes is found almost not to decrease with increasing pillar size. In particular the omnipresent mode pairs F0-F1 and A0-A1 show for all pillar sizes a spacing of about 6 GHz and 2 GHz, respectively.

For most devices, an additional mode E with frequencies...
FIG. 2: Power spectrum density (PSD) versus (a) ascending (P→AP→SF) and (b) descending (SF→AP→P) easy axis applied field for a device of size S, bias current $I = 0.1 \ mA$. (c) corresponding hysteresis loop.

below the mode F0 is observed in the P state in the field region where the resistance departs from its saturation value, indicating increasing non-uniformity of the free layer magnetization (compare e.g. Fig. 2(a) and (c)). As we will see in section V, this mode is likely to belong to excitations localized near the (short) edges of the layer (therefore the label E). Sometimes, second harmonics, such as the mode 2E in Fig. 2(a), are detected. Finally, we label with U (like unidentified) any mode that cannot be assigned immediately to one of the groups F, A, or E.

FIG. 3: Power spectrum density (PSD) versus ascending easy axis field for a device of (a) size M (bias current $I = 0.1 \ mA$) and (c) size L (bias current $I = 0.3 \ mA$). Panels (b), (d): corresponding hysteresis loops, ascending field branch.

2. Size dependent properties

Size S

In the EA spectra of the size-S device in Fig. 2 two FL modes F0, F1, and two SAF modes A0, A1, as well as the edge mode E are observed. After the spin-flop transition, the large angle between the free layer magnetization and the top SAF layer magnetization boosts the experimental sensitivity to all modes (see section V B), rendering visible two additional modes, U1 and U2. From comparison with the measured HA spectra, to be introduced in the next subsection, it follows that the mode U1 must be a SAF mode, while U2 is most likely a FL mode. Note that in Fig. 2(a) the double labeled mode A0-F2 at negative fields has different curvature above and below $-70 \ mT$. Since A0 as an acoustic SAF mode is expected be concave, and to finally reverse its slope at high negative fields, the
change of curvature must be interpreted as the crossing of A0 with a FL mode. At low negative field we observe A0, whereas at higher negative field the FL mode, labeled F2, is observed. The affiliation of the modes F3 and F4 to the FL follows from their slope being much larger than that of A0, which rules them out as potential SAF modes.

In the AP state, several gaps of opening 1 – 1.5 GHz are observed in the positive field branch of mode F0. We will see that this is a consequence of the dipolar coupling between the FL and the SAF leading to anticrossing of the modes. Note that no gap is observed in the negative field branch of F0.

**Size M**

The spectrum in fig. 3(a) for a device of pillar size M shows great resemblance with the spectrum for the size-S device in fig. 2. Again three modes, F0, U2, F1, are observed in the SF region with roughly the same spacing as the modes F0, U2, F1 in fig. 2. Note, however, that all three modes, in particular F0 and U2, have lower overall frequencies than their counterparts in fig. 2. The SAF modes A0 and A1 are spaced by 2 GHz, which is only slightly less than what is commonly observed for size S.

The mode U1 in the P state is either an edge mode or a volume (regular) mode of the FL. However, as U1 does not correspond to the mode U2 (different spacing to F0), the absence of the counterpart of U1 after the SF strongly suggests that U1 is an edge mode, because a regular mode of this intensity in the P state would be visible after the SF as well, since there the sensitivity is even higher than in the P state (see section V B). At low fields, the modes F0 and F2 are strongly deformed, which is most likely a consequence of the non-uniformity of the magnetization in this field region, just like the edge modes.

**Size L**

In the EA spectra of all size-L devices, two FL modes of similar intensity, F0 and F1 in Fig. 3(c), with a spacing of about 5 – 6 GHz are observed in the P state. In the spectrum of a different device than that in Fig. 3(c) (not shown), which has been measured up to 20 GHz, we also observe the counterparts of F0 and F1 after the spin-flop with the same spacing as in the P state, as well as a mode corresponding to U2 of fig. 3(a). The spacing between the modes F0 and F1 is thus approximately the same for pillar size L as for sizes M and S. Similarly, the SAF modes A0, A1, A2, A3 are spaced by about 2 GHz, which is the same as for pillar size M. Note that the mode F1 has gained intensity relative to F0, compared to the size-S spectra. Finally, as for size M, the mode F0 is deformed in the low field region, while a weak additional mode E (edge mode) appears.

---

**B. Eigenexcitations for hard axis applied field**

1. **Size independent properties**

In fig. 4 the HA spectra of the same devices as in figs. 2 and 3 are depicted. The free layer modes have a characteristic W-shape with two minima at about ±70 mT in the lowest modes. Though 70 mT is for all pillar sizes substantially higher than the measured anisotropy fields, the minima are often interpreted as to correspond to the saturation of the free layer magnetization along the HA (cf. also Fig. 1(c),(d)). At zero-field, the modes F0 and F1 show a sharp minimum, which becomes deeper with increasing pillar size, and which is not present in the macrospin HA spectrum (Fig. 1(c)). For some devices (e.g. Fig. 4(c)), we also observe the lowest acoustic mode A0 of the SAF being almost horizontal with a frequency of about 11 GHz at zero field, for all pil-
lar sizes. Contrary to expectation, the spectra are asymmetric w.r.t. zero-field, even at high fields, which cannot be explained merely by a misalignment of the external field with the HA, as we will see in the following paragraph as well as section VI.

2. Size dependent properties

Size S
The HA spectra of size-S devices are characterized by rounded saturation minima of the FL modes at ±70 mT (cf. Fig. 4(a)). The frequency minima of the mode F0 are not zero, but raised to values between 4 and 6 GHz, where the minimum at positive field has sometimes lower and sometimes higher frequency than its counterpart at negative field. Conversely, at high fields, the modes have systematically lower frequency at positive fields (cf. e.g. Fig. 4(a)) - independently of whether the minimum is higher or lower than for negative fields. For some devices, the mode spacing at positive field is significantly reduced compared to negative fields (cf. Fig. 4(a)).

We observe typically four FL modes below 20 GHz with frequencies of about 10, 12, 14.5, and 16 GHz at −140 mT, where the first two modes correspond to F0 and F1 in Fig. 4(a), whereas the last two modes, F2 and F3, are above the frequency range of the figure and therefore not shown. The frequency of the lowest mode F0 is the same for all measured devices of size S, whereas the frequencies of the higher modes F1 to F3 slightly vary from device to device. The mode F0 has much higher intensity than the other modes, where again F3 is slightly more intense than F1 and F2. Note that despite of the higher sensitivity (see section V B) in the HA spectra no mode is observed between F0 and F1, which means that the mode U1 observed in the EA spectrum (Fig. 2) is not a FL mode, but rather a SAF mode, as had already been stated in section III A.

Size M
In the spectra of pillar size M (fig. 4(c)) the saturation minima at ±70 mT in the mode F0 (but not F1) are now deeper than for size S. We shall see in section VII that this is a consequence of the lower dipolar coupling between the FL and the SAF. The minimum at zero-field is considerably sharper than for size S indicating increasing importance of edge domain effects. Finally, the mode spacing has decreased compared to size S.

Size L
In the spectra of pillar size L (Fig. 4(e)) the minima in the mode F0 at ±70 mT reaches - as for size M - markedly lower frequencies than for size S. In difference to size M, now the mode F1 has also pronounced minima at these field values. Both F0 and F1 are strongly deformed in the vicinity of their minima and may even cross each other. The minimum at zero-field has become still deeper, the impact of edge domains now being dominating. The mode spacing between the first two modes has further decreased.

C. History dependence of the mode frequency at zero field

In the previous subsections, we have seen that for increasing pillar size, in both the EA and the HA spectra the modes F0 become progressively deformed in the low field region, which we had attributed to an increasing non-uniformity of the magnetization. At zero field, HA and EA are formally identical, and one should expect the micromagnetic configuration to be the same in both cases. In this section we will see that at least for size L this is not true, but that the micromagnetic configuration at zero field does depend on the field history.

This can be seen by comparing for each pillar size the frequency of the mode F0 at zero field once extracted from the PSD recorded at zero EA field, and once from the PSD measured at zero HA field. Note that on the EA the pillar is in the P state at zero field, if the field was ascending, but in the AP state, if it was descending, whereas on the HA the pillar is always in the AP state at zero field. However, as the two branches of the modes F0 in the EA spectra intersect approximately at zero field, it makes no difference whether on the EA the frequency of F0 is taken from a spectrum measured in the P state or the AP state.

For the three devices of Figs. 2-4 we obtain the following frequencies: for size S 6.55 GHz on the EA and 6.52 GHz on the HA, for size M 5.49 GHz on the EA and 5.49 GHz on the HA, and finally for size L 4.38 GHz on the EA, but 3.75 GHz on the HA. While the frequencies obtained from EA and HA spectra are (within their linewidth) equal for sizes S and M, they differ by more than 0.6 GHz for size L. We conclude that at least for the largest pillar the micromagnetic configuration of the FL at zero field is not the same for EA and HA field history. From the field history invariance of the frequency for sizes S and M it follows that either the micromagnetic configurations are identical, or if they are different, their resonance frequencies are degenerate.

IV. OUTCOMES AND LIMITS OF THE MACROSPIN MODEL

Before making a detailed and rigorous analysis of the field dependence of the modes frequencies in the next section, we start by performing a very simplified analysis of the lowest free layer modes F0. The aim is twofold: motivate the need for a more elaborate analysis by showing quantitative and qualitative limits of conventional modeling done within the macrospin approximation, and obtain approximate starting values for the material parameters.

Approximating the free layer as an isolated rectangular platelet with only shape anisotropy, its ferromagnetic resonance frequency is described by the well-known Kittel law, which for EA applied field (x direction) reads

\[ \omega^2 = \gamma_0^2 [H_{appl}^x + H_k x] [H_{appl}^x + (N_x^n - N_x) M_S], \]

and for HA field (y direction)

\[ \omega^2 = \gamma_0^2 [H_{appl}^y - H_k y] [H_{appl}^y + (N_y^n - N_y) M_S], \]
where $M_S$ is the saturation magnetization of the free layer and $H_k = (N^9 - N^2)M_S$ the shape anisotropy field. Surface anisotropy is negligible in our pillars (see following section).

Applying Kittel fits to the modes $F_0$ in the high field regions of the spectra in Figs. 2-4, allows us to extract $M_S$ and $H_k$ for the different pillar sizes, independently for EA and HA. Using the demagnetizing factors $N^9$, $N^9$, and $N^2$ of Ref. 15, we obtain from the EA spectra the following values for $M_S$ and $H_k$: for size $S$ $\mu_0M_S = 1.29$ T and $\mu_0H_k = 40$ mT, for size $M$ $\mu_0M_S = 1.21$ T and $\mu_0H_k = 21$ mT, and for size $L$ $\mu_0M_S = 1.13$ T and $\mu_0H_k = 8$ mT. Similarly, the HA spectra yield for size $S$ $\mu_0M_S = 1.40$ T and $\mu_0H_k = 50$ mT, for size $M$ $\mu_0M_S = 1.40$ T and $\mu_0H_k = 76$ mT, and for size $L$ $\mu_0M_S = 1.41$ T and $\mu_0H_k = 78$ mT.

It is obvious that the magnetizations and shape anisotropy fields extracted from EA and HA spectra are not consistent with each other. In particular the values for the anisotropy fields are highly contradictory, and in addition, inconsistent with the anisotropy fields extracted from the astroids15 (cf. values in section II), the discrepancies becoming larger with increasing pillar size. Therefore, treating the free layer and the SAF as separate (uncoupled) systems consisting of uniformly magnetized layers is insufficient to describe the eigenexcitations of nanopillars. The next section will be dedicated to a rigorous treatment of spin waves in a coupled three-layer system with lateral confinement.

V. MODEL OF SPIN WAVE MODES IN NANOPILLARS

A. Dipolar-exchange spin waves with quantized wavevectors

1. Eigenexcitations of coupled three-layer system

An MTJ nanopillar consists basically of three confined magnetic layers: the free layer, which will be labeled with the index “1” - reference (top) layer - and “2” - pinned (bottom) layer. The magnetization dynamics in each layer $l \in \{F, 1, 2\}$ of this coupled three-layer system is governed by the Landau-Lifshitz equation.

For small amplitude precessions, the magnetization $\vec{M}_l(\vec{r},t)$ can be decomposed in zeroth order approximation into a time independent uniform (U) equilibrium component $\vec{M}_l^U$ (macrospin, saturation magnetization $M_l$) and a small perpendicular dynamical part $\delta \vec{M}_l^U(\vec{r},t)$. Static non-uniformities of the equilibrium magnetization will be discussed in section V.B. The dynamical component $\delta \vec{M}_l^U(\vec{r},t)$ can be approximated as a sum of plane spin waves,

$$\delta \vec{M}_l^U(\vec{r},t) = \Re \sum_{\vec{k}} \delta \vec{M}_l^U(\vec{k}) e^{i\vec{k} \cdot \vec{r} - i\omega t},$$

where the wavevectors $\vec{k} = (k_x, k_y, 0)$ of the partial waves are quantized due to the spatial confinement of the layers. The out-of-plane component $k_z$ is zero for all modes in the experimental scope due to the very small layer thicknesses of 2 – 3 nm. The quantization of the in-plane components will be discussed in detail later on. The frequencies $\omega$ of the partial waves are the eigenfrequencies of the three-layer system.

In the effective fields acting on the magnetizations the following interactions have been taken into account: the applied field $H_{appl}$, the exchange bias field acting on the bottom layer of the SAF (coupling constant $J^{eb}$), the interlayer exchange coupling of the SAF layers (coupling constant $J^{int}$), and the (intralayer) exchange interaction in each layer (exchange stiffness constant $A_l$), as well as the demagnetizing fields and mutual dipolar coupling of the layers. For the demagnetizing fields we use the standard tensor expression for uniformly magnetized ellipsoidal bodies, where the diagonal components of the diagonal demagnetizing tensors $N_l$ are the demagnetizing factors $N_{xl}^T$, $N_{yl}^T$, $N_{zl}^T$ of the rectangular layers. The dipolar coupling fields are given by analog expressions where the (self-)demagnetizing tensors of trace 0 are replaced by the mutual demagnetizing tensors $N_{ml}^{\parallel}$ of trace 0 ($l, m \in \{F, 1, 2\}$, $l \neq m$).

2. Quantization of in-plane wavevector

The in-plane components $k_x$, $k_y$ of the wavevector are determined by the boundary conditions (BC) imposed on the dynamical magnetization (1) at the lateral layer boundaries $x = \pm L_x/2$ and $y = \pm L_y/2$. For simplicity we will consider the $x$-component (along the long edge of the rectangle) as an example, where any of the following statements equally hold for the $y$-component with $x$ and $y$ permuted. For the $x$-component the BC read:

$$\frac{\partial}{\partial \xi_x} \delta \vec{M}_l^U(\xi_x, \xi_y) \pm d_x^{\pm} (\delta \vec{M}_l^U(\xi_x, \xi_y))_{\xi_x = \pm \frac{L_x}{2}} = 0.$$
where \( \xi_x = x/L_x \). Eq. (2) is a modified version of the effective BC derived by Guslienko et al.\(^9\) for thin magnetic stripes. In difference to Ref. 9 we allow for different pinning parameters \( \delta_x^0 \) and \( \delta_x^2 \) at opposite boundaries \( x = \pm L_x/2 \) to account for potential asymmetries in the pinning expected from a real device. Moreover, instead of using the analytic expression (5) in Ref. 9 to calculate the (dimensionless) pinning parameters, we will extract approximate values for \( \delta_x^0 \) from the experimental spectra (see section VI).

With the spatial dependence of \( \delta \dot{M}_x^U \) (1) rewritten as

\[
\Re e^{i \Phi} = \sin(k_y x + \phi_y) \sin(k_x y + \phi_x),
\]
the BC (2) for the wavevector component \( k_x \) and the phase \( \phi_x \) the two conditions

\[
\mp k_x L_x \cot \left( \pm k_x \frac{L_x}{2} + \phi_x \right) = d_x^\pm.
\]

It is convenient to express \( k_x L_x \) in the argument of the cotangent as multiples of \( \pi \), thus defining the - in general nonintegral - mode numbers

\[
n_x = k_x \frac{L_x}{\pi}.
\]

For symmetric pinning, \( d_x^+ = d_x^- = d_x \), it follows from (4) that the cotangent has to be antisymmetric, yielding \( \phi_x^* = \pi/2 \) or \( \phi_x^0 = 0 \), i.e. symmetric or antisymmetric wavefunctions (3). In the limiting case of totally unpinned BC, \( d_x = 0 \), the mode numbers \( n_x^0 \) are integers, starting at 0, and the corresponding wavefunctions alter between symmetric and antisymmetric for successive mode numbers, starting with symmetric, such that there are always antinodes at both boundaries.

For finite values \( d_x > 0 \) of the pinning, the mode numbers \( n_x \) are no longer integers. Plotting \( n_x \) versus \( d_x \) by means of eqs. (4) and (5) shows that with increasing \( d_x \), the deviations \( \Delta n_x \) of \( n_x \) from the corresponding integral values \( n_x^0 \) of the unpinned case increase continuously from \( \Delta n_x = 0 \) for \( d_x = 0 \) (unpinned) to \( \Delta n_x = 1 \) for \( d_x = \infty \) (totally pinned). Therefore, the mode numbers for total pinning, \( n_x^\infty = n_x^0 + 1 \), are integers again. For a fixed intermediate value \( d_x \), the deviation \( \Delta n_x \) of the mode number \( n_x \) from the corresponding integral mode number \( n_x^0 \) is found to rapidly decrease with increasing \( n_x^0 \). For a given pinning, the mode numbers are therefore no independent variables: if one mode number (e.g. that of the lowest mode) has been fixed, all other mode numbers are fixed, too.

In case of slightly asymmetric pinning, \( d_x^+ \neq d_x^- \), the phase \( \phi_x \) differs from the values \( \phi_x^a \) by a small phase shift \( \Delta \phi_x \), such that the wavefunctions are no longer totally symmetric or antisymmetric. In this case, the mode numbers \( n_x \) are necessarily non-integral. In the hypothetical case of totally asymmetric pinning, \( d_x^+ = 0 \) and \( d_x^- = \infty \) (or vice versa), \( \Delta n_x = 0.5 \) and \( \Delta \phi_x = \Delta \phi_x^a = \pi/4 \). For arbitrary pinning, \( \Delta \phi_x \) is an unknown function of \( d_x^0 \) and \( n_x \).

With the spatial dependence of \( \delta \dot{M}_x^U \) (1) rewritten as

\[
\Re e^{i \Phi} = \sin(k_x x + \phi_x) \sin(k_y y + \phi_y),
\]

the BC (2) for the wavevector component \( k_x \) and the phase \( \phi_x \) the two conditions

\[
\mp k_x L_x \cot \left( \pm k_x \frac{L_x}{2} + \phi_x \right) = d_x^\pm.
\]

It is convenient to express \( k_x L_x \) in the argument of the cotangent as multiples of \( \pi \), thus defining the - in general nonintegral - mode numbers

\[
n_x = k_x \frac{L_x}{\pi}.
\]

For symmetric pinning, \( d_x^+ = d_x^- = d_x \), it follows from (4) that the cotangent has to be antisymmetric, yielding \( \phi_x^* = \pi/2 \) or \( \phi_x^0 = 0 \), i.e. symmetric or antisymmetric wavefunctions (3). In the limiting case of totally unpinned BC, \( d_x = 0 \), the mode numbers \( n_x^0 \) are integers, starting at 0, and the corresponding wavefunctions alter between symmetric and antisymmetric for successive mode numbers, starting with symmetric, such that there are always antinodes at both boundaries.

For finite values \( d_x > 0 \) of the pinning, the mode numbers \( n_x \) are no longer integers. Plotting \( n_x \) versus \( d_x \) by means of eqs. (4) and (5) shows that with increasing \( d_x \), the deviations \( \Delta n_x \) of \( n_x \) from the corresponding integral values \( n_x^0 \) of the unpinned case increase continuously from \( \Delta n_x = 0 \) for \( d_x = 0 \) (unpinned) to \( \Delta n_x = 1 \) for \( d_x = \infty \) (totally pinned). Therefore, the mode numbers for total pinning, \( n_x^\infty = n_x^0 + 1 \), are integers again. For a fixed intermediate value \( d_x \), the deviation \( \Delta n_x \) of the mode number \( n_x \) from the corresponding integral mode number \( n_x^0 \) is found to rapidly decrease with increasing \( n_x^0 \). For a given pinning, the mode numbers are therefore no independent variables: if one mode number (e.g. that of the lowest mode) has been fixed, all other mode numbers are fixed, too.

In case of slightly asymmetric pinning, \( d_x^+ \neq d_x^- \), the phase \( \phi_x \) differs from the values \( \phi_x^a \) by a small phase shift \( \Delta \phi_x \), such that the wavefunctions are no longer totally symmetric or antisymmetric. In this case, the mode numbers \( n_x \) are necessarily non-integral. In the hypothetical case of totally asymmetric pinning, \( d_x^+ = 0 \) and \( d_x^- = \infty \) (or vice versa), \( \Delta n_x = 0.5 \) and \( \Delta \phi_x = \Delta \phi_x^a = \pi/4 \). For arbitrary pinning, \( \Delta \phi_x \) is an unknown function of \( d_x^0 \) and \( n_x \).

### Table 1: Dependence of the resistance noise (7) on the mode character and the micromagnetic configuration.

<table>
<thead>
<tr>
<th>Field region</th>
<th>Field strength</th>
<th>Mode number criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy axis</td>
<td>( P ) state</td>
<td></td>
</tr>
<tr>
<td>Low H</td>
<td>( M_1 \Delta \theta )</td>
<td></td>
</tr>
<tr>
<td>High H</td>
<td>( M_1 \Delta \theta + \delta \dot{M}_x^U )</td>
<td></td>
</tr>
<tr>
<td>AP state</td>
<td>Medium H</td>
<td>( M_1 \Delta \theta )</td>
</tr>
<tr>
<td>High H</td>
<td>( M_1 \Delta \theta + \delta \dot{M}_x^U )</td>
<td></td>
</tr>
<tr>
<td>Hard axis</td>
<td>( H &gt; H_{SF} )</td>
<td>( M_1 \sin \theta_0 )</td>
</tr>
</tbody>
</table>

The pinning is determined by two competing interactions: the dipolar interaction favoring pinned BC, by tempting to avoid a dynamical magnetization component perpendicular to the boundaries, and the exchange interaction favoring unpinned BC, by trying to align the local magnetic moments. Therefore, the smaller the lateral dimensions of a layer, the higher will be the impact of the exchange interaction. Consequently, the pinning is expected to be larger for pillar size \( L \) than for pillar size \( S \). For the same reason, the pinning should also be larger for the \( x \)-direction than and for the \( y \)-direction of the same device, as the dimensions \( L_x, L_y \) differ by a factor 2.

### B. Expected experimental sensitivity

#### 1. Formulation of the problem

As described in section II, the experimental spin wave spectra are obtained by measuring the voltage noise, which, for small bias currents, is proportional to the MR noise generated by the spin waves in the free layer (FL) and the reference layer (RL). To be more precise, we measure the spatial average of the MR noise over these two layers. For the sake of simplicity, we will derive the expected MR noise for excitations in...
the FL, where the analog expressions for the RL are obtained by permuting the indices F and 1.

The resistance noise signature of a partial spin wave \( \delta \vec{M}_{F,1}(\vec{r}, \vec{k}) \) is defined as the projection of \( \delta \vec{M}_{F,1}(\vec{r}) \) onto the equilibrium magnetization \( \vec{M}_{1}(\vec{r}) \), resulting in the spatial average of the edge domain contributions to the equilibrium magnetization. In the dynamical part, however, we still take into account only the dynamics \( \delta \vec{M} \) perpendicular to the macrospin \( \vec{M}_{1} \) components, which are perpendicular to the macrospin \( \vec{M}_{1} \).

In the dynamical part, however, we still take into account only the dynamics \( \delta \vec{M} \) of the vector \( \delta \vec{M} \) perpendicular to the macrospin \( \vec{M}_{1} \) components, which are perpendicular to the macrospin \( \vec{M}_{1} \) and \( \vec{M}_{1} \) of the two layers. The resistance variation (6) thus becomes

\[
\delta R_F(\vec{k}, \theta) = \overline{M_{F}^{\text{dyn}}(\vec{k}) \overline{M_{1}^{\text{eq}}(\theta)}}
\]

where

\[
\overline{M_{F}^{\text{dyn}}(\vec{k})} = \overline{\delta \vec{M}_{F}(\vec{k})} = \frac{1}{V_F} \int_{F} \Re \vec{k} \cdot \delta \vec{M}_{F}(\vec{r}) \, d\vec{r}
\]

is the spatial average over the FL spin wave, and

\[
\overline{M_{1}^{\text{eq}}(\theta)} = M_{1} \sin \theta + \overline{\delta \vec{M}_{1}^{\text{eq}}(\vec{k})} \cos \theta,
\]

the projection of the spatially averaged equilibrium magnetization of the FL on the amplitude vector \( \delta \vec{M}_{1}(\vec{k}) \) of the spin wave. \( \overline{\delta \vec{M}_{1}^{\text{eq}}(\vec{r})} \) is the modulus of the vector \( \delta \vec{M}_{1}^{\text{eq}} = \frac{1}{V_1} \int_{F} \Re \vec{k} \cdot \delta \vec{M}_{1}^{\text{eq}}(\vec{r}) \, d\vec{r} \) describing the spatial average of the edge domain contributions to the equilibrium magnetization.

Being a function of the macrospin orientation as well as of the spatial distribution of potential edge domains, the factor \( \overline{M_{1}^{\text{eq}}(\theta)} \) describes the dependence of the MR noise on the micromagnetic configuration of the layer \( l \in \{ F, 1 \} \) and is therefore sensitive to direction and strength of the external field. \( \overline{M_{1}^{\text{eq}}(\vec{k})} \), on the other hand, depends on the wavefunction of the spin waves and is consequently sensitive to the mode number and symmetry properties of the eigenexcitation. In the following, we will investigate the dependence of the measured MR noise on the mode character and on the micromagnetic configuration in more detail.

2. Mode character dependence of sensitivity

By means of eqs. (3) and (5) the spatial average of the wavefunction over the layer volume \( V_l \) in eq. (8) is easily evaluated as

\[
\frac{1}{V_l} \int \Re \vec{k} \cdot \delta \vec{M}_{F}(\vec{r}) \, d\vec{r} = \nu_x(n_x, \phi_x) \nu_y(n_y, \phi_y),
\]

where \( \nu_x(n_x, \phi_x) \) and \( \nu_y(n_y, \phi_y) \), given by an analog expression, are the 1D integrals of the two sine functions in (3) over the layer dimension in direction \( x, y \). Again we consider as an example the \( x \)-component.

We decompose the mode numbers \( n_x \) as above into the integral part \( n_x^0 \) of the unpinned case and the deviation \( 0 \leq \Delta n_x \leq 1 \) from this value due to pinning. Similarly, the phase \( \phi_x \) is decomposed into the phase \( \phi_x^0 = \pi/2 + n_x^0 \cdot \pi/2 \) for symmetric pinning and a small phase shift \( \Delta \phi_x \) in case of slightly asymmetric pinning. \( \nu_x(n_x, \phi_x) \) can then be expanded in \( \Delta \phi_x \ll 1 \) and either \( \Delta n_x \ll 1 \) (weak pinning) or \( \Delta n_x \approx 1 \) (strong pinning). The result as a function of the integral mode number \( n_x^0 \) of the unpinned case is shown in Tab. I(a). Note that for strong pinning the natural reference mode numbers are the mode numbers \( n_x^0 \), \( n_x^0 + 1 \) of total pinning, where odd (even) \( n_x^0 \) in the table correspond to even (odd) \( n_x^0 \).

As can be seen from Tab. I(a), for zero pinning \( \Delta n_x = \Delta \phi_x = 0 \), \( n_x^0 = 0 \) is the only visible mode. In the presence of pinning, the higher modes \( n_x^0 \geq 1 \) begin to appear: for symmetric pinning \( \Delta n_x > 0, \Delta \phi_x = 0 \) only those with even mode numbers \( n_x^0 \) (symmetric wavefunctions), in case of asymmetric pinning \( \Delta n_x > 0, \Delta \phi_x \neq 0 \) also those with odd \( n_x^0 \) (antisymmetric wavefunctions).

For weak pinning \( \Delta n_x \ll 1 \), \( \nu_x(n_x, \phi_x) \) is for all higher modes \( n_x^0 \geq 1 \) of first order in a small quantity: in \( \Delta n_x \) for even \( n_x^0 \), in \( \Delta \phi_x \) for odd \( n_x^0 \). It follows that \( \nu_x(n_x, \phi_x) \) decreases rapidly with increasing mode number: firstly because of the factor \( 1/n_x \), secondly because both \( \Delta n_x \) and \( \Delta \phi_x \) become smaller as \( n_x^0 \) increases (cf. previous subsection), \( \Delta n_x \) and \( \Delta \phi_x \) are of the same order of magnitude (typically of the order 0.1), if the asymmetry in the pinning at opposite boundaries is comparable to the overall pinning, which is realistic for weak pinning, since there the asymmetry - as an absolute quantity - has a higher relative impact.

For strong pinning \( \Delta n_x \approx 1 \), \( \nu_x(n_x, \phi_x) \) for odd \( n_x^0 \) (even \( n_x^0 \)) becomes second order in \( (1 - \Delta n_x) \) and \( \Delta \phi_x \), whereas for even \( n_x^0 \) (odd \( n_x^0 \)) it is now a zeroth order quantity like the lowest mode \( n_x^0 = 0 \) (\( n_x^0 = 1 \)). For all modes, \( \nu_x(n_x, \phi_x) \) decreases with increasing mode number as \( 1/n_x \); the decrease of \( \Delta n_x \) and \( \Delta \phi_x \), however, does not affect modes with even \( n_x^0 \) and is partly compensated for odd \( n_x^0 \), as the factor \( (1 - \Delta n_x) \) increases with decreasing \( \Delta n_x \).
Since the mode intensity is proportional to the square of $M_{\text{sym}}^{E}(\hat{k})$ (8), only those modes $(n_x, n_y)$ are expected to be detected, for which the product $\nu_x(n_x, \phi_x) \nu_y(n_y, \phi_y)$ (10) is at most of first order in $\Delta \phi_x$ or $\Delta n_x$.

For weak pinning, we therefore expect to observe in addition to the quasi-uniform mode close to $(0,0)$ higher modes with mode numbers close to $(1,0), (0,1), (2,0), (0,2),$ and possibly $(3,0)$. Their intensities are thereby reduced compared to that of $(0,0)$ by factors scaling quadratically with $(3,0)$. Their intensities are thereby reduced compared to that of $(1,1)$ except for the reduction factor $1/n^2_{x,y}$. For strong pinning, the lowest mode is expected to be close to $(1,1)$. Higher modes close to $(3,1), (1,3),$ and $(5,1)$ will have intensities comparable to that of $(1,1)$ except for the reduction factor $1/n^2_{x,y}$.

3. **Micromagnetic configuration dependence of sensitivity**

The micromagnetic factor $M_{\text{sym}}^{E}(\theta)$ (9) for layer $l \in \{F, L\}$ is a function of the angle $\theta$ between the macrospins of the FL and the RL, as well as of the spatial distribution of potential edge domains underlying the quantity $\delta M^{E}$. Both $\theta$ and $\delta M^{E}$ strongly depend on direction and strength of the external field, which is why in the following we will distinguish between easy and hard axis applied field and identify characteristic field regions of distinct values $\theta$ and $\delta M^{E}$.

As a first step, we decompose $\theta$ into a dominating part $\theta_0$, denoting the angle between the two macrospins in the case that the external field $H_{\text{appl}}$ as well as the exchange bias field $H_{\text{eb}}$ are perfectly aligned with the symmetry axes of the layer, and a small deviation $\Delta \theta$ from the ideal angle, caused by a slight misalignment of $H_{\text{appl}}$ or $H_{\text{eb}}$ with the layer axes as expected from a real device. For EA applied field, $\theta_0 = 0$ in the P state, $\theta_0 = \pi$ in the AP state, and $0 < \theta_0 < \pi/2$ after the SF. For HA field, $\theta_0$ decreases continuously from $\pi$ at zero field to a value close to $\pi/2$ at the saturation field of the FL, and finally towards zero as the RL magnetization continues to tilt towards the HA. $M_{\text{sym}}^{E}(\theta)$ can then be expanded in $\Delta \theta$ where we only keep the leading order terms; the result for the different field regions is summarized in Tab. I(b).

On the EA, the presence of edge domains can be deduced from the hysteresis loops: e.g. for ascending field, the resistance increases continuously in the P state when the field approaches the switching field to the AP state. The reason for this behavior are increasing nonuniformities of the FL magnetization, the latter being subject to an increasing antiparallel effective field consisting of the dipolar coupling field created by the RL, the (self)demagnetizing field, and the external field (as soon as it becomes positive). In the AP state, the resistance starts to decrease already long before the SF, which can be attributed to increasing nonuniformities of the RL magnetization, which is pointing antiparallel to the high external field. Therefore, non-negligible contributions from edge domains are expected for the FL at low fields in the P state, and for the RL at high fields in the AP state, near the SF transition, as marked in the table.

However, note that the spatial average $\delta M^{E}$ over the edge domains is zero if all components of $\delta M^{E}$ are antisymmetric w.r.t. the symmetry axes of the layer.

On the HA, edge domain contributions $\delta M^{E}$ are negligible compared to the contribution $M_1 \sin \theta_0$ of the macrospins, except for zero field, where the macrospins are antiparallel.

As can be seen from Tab. I(b), large values of $M_{\text{sym}}^{E}(\theta)$ and hence high sensitivity for both FL and SAF modes can be expected on the HA at any finite field value and on the EA after the SF. Weak higher modes will therefore be visible, if at all, in these field regions. On the EA, at fields below the SF, modes become visible only through the misalignment $\Delta \theta$ of the macrospins or through edge domains. Slightly increased sensitivity is therefore expected for FL modes in the AP state just before the SF, and for SAF modes in the P state at low fields. The presence of the edge domains thereby entails the appearance of edge modes in the spectra.

### VI. Extractions of Material Parameters

In section V A, we have derived the mode frequencies $\omega_0$ as a function of the material parameters $M_1$, $A_1$, $J_{\text{eb}}$, $J_{\text{int}}$, the geometry parameters $(L_x, L_y)$, $\{N_1^x, N_1^y, N_1^z\}$, and $(N_{\text{ml}}^x, N_{\text{ml}}^y, N_{\text{ml}}^z)$, as well as the mode numbers $(n_x, n_y)$. In this section, we will finally extract these parameters from the experimental spectra. Since the model is based on the assumption of a uniform equilibrium magnetization, we will use the spectra of the smallest pillar size $S$ (Figs. 2 and 4(a)), for which the non-uniformities of the magnetization had been found to be

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturation</td>
<td>$1.27 \text{T}$</td>
</tr>
<tr>
<td>Magnetization</td>
<td>$1.37 \text{T}$</td>
</tr>
<tr>
<td>Exchange</td>
<td>$16.4 \times 10^{-12} \text{J/m}$</td>
</tr>
<tr>
<td>Stiffness</td>
<td>$12.3 \times 10^{-12} \text{J/m}$</td>
</tr>
<tr>
<td>Exchange Bias</td>
<td>$3.5 \times 10^{-14} \text{J/m}^2$</td>
</tr>
<tr>
<td>Demagnetizing Factors</td>
<td>$0.035, 0.055, 0.9$</td>
</tr>
<tr>
<td>Dipolar Coupling Constants</td>
<td>$(0.005, 0.009, -0.014)$</td>
</tr>
<tr>
<td>Mode Numbers</td>
<td>$(0.4, 0.2)$</td>
</tr>
</tbody>
</table>

Table II: Material and geometry parameters used to calculate the spectra and hysteresis loops in Fig. 5 belonging to the device of pillar size $S$ in Figs. 2 and 4(a). Error bars for the parameters are given in the text.
As a matter of fact, not all of the above quantities are free
input parameters to the model. Given the (approximate) layer
dimensions $L_x, L_y, L_z$, the demagnetizing factors $N_x^0, N_y^0, N_z^0$ can be calculated using e.g. OOMMF simulations, where we find $N_z^0 \approx 1 - (N_x^0 + N_y^0)$ and $N_y^0/N_z^0 \approx L_x/L_y$ as should be expected. Using the formulae in Ref. 21 it can be shown that the dipolar coupling constants obey similar relations, $N_{mn}^0/N_{ml}^0 = L_x/L_y$ and $N_{mn}^0 = - (N_{ml}^0 + N_{nm}^0)$. Finally, for symmetry reasons, $N_{nml} = N_{nlm}$. The remaining components $N_{F1}^0, N_{F2}^0$ and $N_{E}^0$ are kept as free parameters to be extracted from the experiment, although they can be calculated by means of Ref. 21. We will compare the calculated and extracted values later on.

On the basis of previous measurements on thin films, the number of free parameters can be further reduced: From comparison of the results of Refs. 14 and 15 it can be seen that the effective magnetization of annealed CoFeB thin films (of similar ratio Co:Fe) shows no particular dependence on the film thickness in the range from 40 to 3 nm. We may therefore assume that the FL and the RL have equal magnetizations, $M_I = M_F$, despite of their different thicknesses. Moreover, we expect the layer magnetizations in the pillar to be reduced for all layers by the same (relative) amount w.r.t. the thin film saturation magnetization $M_{sat}$. The effective saturation magnetization of annealed CoFeB thin films (of similar composition) has been reported to be in the interval $0.8 \pm 0.1$ T.

In this paragraph, we list as an orientation literature values for the above parameters. As thin film exchange stiffness constants we use the values of the 40 nm CoFeB and CoFe films in Ref. 14: $A_{film}^{CoFe} = 28.4 \times 10^{-12} \text{J/m}$ and $A_{film}^{CoFe} = 27.5 \times 10^{-12} \text{J/m}$. The magnetizations of CoFe and annealed CoFeB depend on the percentage of Fe in Co. The bulk value for both Co$_{70}$Fe$_{30}$ and Co$_{75}$Fe$_{25}$ (corresponding to Co$_{60}$Fe$_{20}$B$_{20}$) is $(2.2 \pm 0.1)$ T. The magnetizations of annealed CoFeB thin films of this composition have been measured to be $\mu_0 M_{film}^{CoFe} = (1.8 \pm 0.1)$ T. The thin film value for the CoFe layer, as part of the annealed MTJ stack, is expected to be in the interval $\mu_0 M_{film}^{CoFe} = (2.0 \pm 0.2)$ T. The exchange bias field in a Co$_{60}$Fe$_{20}$B$_{20}$ (5 nm)/PdMn (20 nm) system has been measured to be $\mu_0 H^{eb} \approx 67 \text{ mT}$, which corresponds to an exchange bias energy of $J^{eb} = 4.5 \times 10^{-3} \text{J/m}^2$.}

**A. Reduction of number of free parameters**

FIG. 5: Calculated mode frequencies versus external magnetic field along easy axis (a) and hard axis (c) for the device of pillar size $S$ in Fig. 2 and 4(a). Panels (b) and (d) show the corresponding calculated hysteresis loops. In each case the field was tilted by $+2^\circ$ w.r.t. the symmetry axis. The parameters used to calculate the spectra are given in Tab. II. The indices $m,n$ in the mode labels $f_{mn}$ and $a_{mn}$ correspond to the following mode numbers $(n_x, n_y)$:

- 00 = (0,4,0,2), 10 = (1,15,0,2), 01 = (0,4,1,06),
- 20 = (2,08,0,2). In panel (a), filled symbols are used for ascending field (P→AP→SF) and half-filled symbols for descending field (SF→AP→P). The dashed vertical lines in panel (c) mark the borders of the experimental hard axis spectrum in Fig. 4(a).

B. Literature values

In this paragraph, we list as an orientation literature values for the above parameters. As thin film exchange stiffness constants we use the values of the 40 nm CoFeB and CoFe films in Ref. 14: $A_{film}^{CoFe} = 28.4 \times 10^{-12} \text{J/m}$ and $A_{film}^{CoFe} = 27.5 \times 10^{-12} \text{J/m}$. The magnetizations of CoFe and annealed CoFeB depend on the percentage of Fe in Co. The bulk value for both Co$_{70}$Fe$_{30}$ and Co$_{75}$Fe$_{25}$ (corresponding to Co$_{60}$Fe$_{20}$B$_{20}$) is $(2.2 \pm 0.1)$ T. The magnetizations of annealed CoFeB thin films of this composition have been measured to be $\mu_0 M_{film}^{CoFe} = (1.8 \pm 0.1)$ T.
two rectangular layers of equal thicknesses. As in our pillars the FL has a different thickness than the two SAF layers, only \( N_{12} \) may be calculated directly, yielding \( N_{12} = 0.016 \). The dipolar coupling constants \( N_{F1} \) and \( N_{F2} \) involving the FL can only be estimated as the mean value of the constants calculated for two 3 nm thick layers and for two 2 nm thick layers, from which we obtain \( N_{F1} \approx 0.018 \) and \( N_{F2} \approx 0.013 \) (maximum deviation \( \pm 0.003 \)).

C. Regression method

The parameters \( M_F, A_F, J^{int}, J^{hh}, N^x_{F1}, N^x_{F2}, N^{x}_{12} \), and \( (n_x, n_y) \) are obtained by adjusting the calculated modes and hysteretic loops to the corresponding experimental data (Figs. 2 and 4(a),(b)).

\( M_F \) is determined by the modes F0 on the EA with a weak dependence on the chosen mode numbers of F0 (see discussion below). \( M_1 \) and \( M_2 \) cannot be extracted directly, but depend entirely on the above assumptions. \( J^{int} \) and \( J^{hh} \) can be extracted from the modes A0 and are strongly affected by the actual values of \( M_1 \) and \( M_2 \). \( N^x_{F1} \) and \( N^x_{F2} \) follow from the gap openings in the mode F0 on the EA and the shift of the EA hysteresis loop to negative fields: Both \( N^x_{F1} \) and \( N^x_{F2} \) have to be smaller than 0.006, because otherwise the gap opening exceeds 1.5 GHz, while the difference \( N^x_{F1} - N^x_{F2} \) must be at least 0.004 to assure a shift of the hysteresis loop of minimum 5 mT.

The exchange stiffness constant \( A_F \) and the mode numbers \( (n_x, n_y) \) cannot be extracted separately, since they enter the effective field (and consequently the frequencies) only as a product. The BC in the pillar being unknown, the lowest mode can have any mode numbers between (0,0) (unpinned BC) and (1,1) (totally pinned BC). However, in order to adjust F0 on the EA with the mode (1,1), an exchange stiffness of \( A_F \approx 1/20 \cdot A_F^{film} \) would be needed, which is unreasonably low seen that \( M_F \approx 2/3 \cdot M_F^{film} \). In addition, on the HA, a discrepancy between calculated and measured mode F0 of 1.5 GHz or more is observed even at high fields. Similarly, fitting F0 with the mode (0.5,0.5), which one might consider as the limit between strong and weak pinning, still requires \( A_F < 1/5 \cdot A_F^{film} \) with a minimum deviation of the modes of \( > 0.6 \) GHz. Strong pinning can therefore be excluded in our pillars; the mode numbers of F0 must be well below (0.5,0.5) in order to obtain satisfactory agreement between calculated and measured mode frequencies and values for \( A_F \) in a physically reasonable range.

If, on the hand, we assume totally unpinned BC - fitting F0 with (0,0) and the higher modes with (1,0), (0,1), and (2,0) - we get \( A_F \approx 2/3 \cdot A_F^{film} \), i.e. \( A_F \propto M_F \). However, in section V B we have seen that for totally unpinned BC, none of the higher modes should be visible in the measured spectrum. Since we clearly observe three FL modes above F0 (cf. section III B), the case of zero pinning must be excluded, too, in spite of the good agreement in the mode frequencies. Consequently, there must be some weak pinning at the lateral boundaries of our pillars. The appearance of the modes close to (1,0) and (0,1) further suggests a slight asymmetry of the pinning as discussed in section V B. Finally, \( n_y \) is likely to be slightly larger than \( n_y \) due to the different pillar dimensions in \( x \) - and \( y \)-direction (cf. section V A). This assumption is corroborated by the fact that the mode close to (0,1) is obviously weaker than that close to (1,0), the latter having sufficient intensity to be observed even in the EA spectrum, while the former is invisible.

D. Best parameter estimate

Best overall agreement is obtained for the parameters in Tab. II. The corresponding calculated spectra and hysteresis loops are shown in Fig. 5. In the following, we will discuss the “technical” reliability of the different parameter values. Their physical relevance will be discussed in the next section.

For the lateral dimensions we have used the measured mean values, \( L_x = 100 \) nm and \( L_y = 60 \) nm. \( L_x, L_y \) may vary by 10 nm, which changes the geometry related parameters accordingly. The mode numbers of F0 can take on values between (0.2, 0.1) and (0.4, 0.2) in order to ensure satisfactory agreement in frequency and a reasonable value for \( A_F \). The mode numbers in the table are thus maximum values, which we consider to describe best the observed mode intensities. The value of \( \mu_0 M_F \) is found to be between 1.27 T (max. mode numbers) and 1.3 T (min. mode numbers). \( A_F \) is expected to lie in the interval \((17 \pm 1) \times 10^{-12} \) J/m. Under the above assumptions the same holds true for \( M_1 \) and \( A_1 \). The value of \( M_2 \) and consequently \( A_2 \) depends additionally on the actual thin film value \( M_2^{film} \) of the CoFe layer; accordingly, \( \mu_0 M_2 \) is expected to be contained in the interval \((1.4 \pm 0.1) \) T, and \( A_2 \) in \((16 \pm 3) \times 10^{-12} \) J/m. Finally, depending on \( M_2 \), \( J^{hh} \) is found to be between \( 3.2 \times 10^{-4} \) J/m² and \( 4.0 \times 10^{-4} \) J/m², and \( J^{int} \) between \( -2.8 \times 10^{-4} \) J/m² and \( -3.1 \times 10^{-4} \) J/m². \( N^x_{F1} = 0.005 \) and \( N^x_{F2} = 0.001 \) are uniquely determined by the experimental constraints with a maximum deviation of \( \pm 0.001 \). Since for the calculated constants \( N^y_{12} \approx N^y_F \), we have set \( N^y_{12} = 0.005 \) as well.

From comparison of Figs. 5(a) and 2, it can be seen that at high and medium fields the measured and calculated EA spectra are in quantitative agreement for the chosen parameters. Minor deviations occur in the higher SAF modes, the shape of the FL modes near the SF, and the position of the gaps in the mode F0. The HA spectra, Figs. 5(c) and 4(a), show quantitative agreement at medium and high negative fields only. The discrepancy at positive fields is due to the systematic high-field asymmetry of the experimental HA spectra described in section III B. This asymmetry cannot be accounted for by a tilting of the field w.r.t. to the HA: The tilting of \( +2^\circ \), used to calculate the spectra and hysteresis loops in Fig. 5, has hardly any impact on the mode frequency in the high field region, while it is sufficient to correctly reproduce the asymmetry of the minima in the FL modes at medium HA fields. More-
over, it already causes a measurable asymmetry of the hysteresis loop, the resistance at positive fields being lower than at negative fields, in contrast to the measured hysteresis loop (Fig. 4(b)), where the resistance is found be higher at positive than at negative fields - for the same tendency in the mode frequencies. The asymmetry in the HA spectra must therefore be due to intrinsic imperfections of the pillar, such as a misalignment of the exchange bias field with the symmetry axes of the layers. Note that asymmetries in the pillar shape, being more or less random, would not explain the systematic character of the asymmetry. In the zero-field region, the differences between calculated and experimental HA spectra become substantial, as the measured modes develop pronounced minima, whereas the model predicts a local frequency maximum. On the EA, the low field region of the P state is characterized by the appearance of the low-frequency modes $E$ in the experimental spectra, which are not present in the calculated spectra, and which are accompanied by a significant difference in the measured and calculated mode frequencies of all higher modes. Finally, note the good agreement of the calculated hysteresis loops in Fig. 5(b), (d) with the experiment, in contrast to the loops in Fig. 1(b), (d) calculated in the standard macrospin approximation.

VII. DISCUSSION

In the previous section, we have modeled the spin wave spectra of MTJ nanopillars as eigenexcitations of a coupled three-layer system with lateral confinement. In this section, we will see, which properties of the experimental spectra can be explained in the scope of this analytical model, and which cannot. First, we will discuss the material parameters of the pillar extracted from the high field regions of the spin wave spectra. Thereafter, the low-field anomaly of the spectra and its relevance for applications will be discussed. Finally, we will summarize the properties of the experimental spectra, which are beyond the approximations of our model, including the pillar size dependence.

A. Material and geometry parameters

In this subsection, we will discuss the physical relevance of the extracted parameter values of the pillar given in Tab. II. With $1.27T$ the saturation magnetization of the $\Co_{00}\Fe_{20}B_{20}$ layers is significantly reduced compared to the thin film value of $(1.8 \pm 0.1)T$, or the bulk value for the underlying $\Co_{75}\Fe_{25}$ of $(2.2 \pm 0.1)T$. Note that this value of $M_S$ is valid for both field orientations, in contrast to the Kittel fits in section IV, where different values had been obtained on easy and hard axis. A reduction of the magnetization in nanopillars has already been observed in previous studies on pillar devices. Three scenarios are usually suggested to account for this phenomenon: process-induced damages, current-induced heating, or a nonlinear change of the frequency with high mode amplitude. As we work with low bias current, current induced heating can be excluded in our case. Similarly, since spin-torque induced auto-oscillations in our samples occur typically for currents above $1.6\ mA$ for size L, high amplitude nonlinear effects as possible cause can be rejected, too. Therefore, some sort of process damage, such as ion implantation, diffusion, intrinsic chemical modifications or interface effects, must be at the origin of the magnetization reduction, whose further investigation exceeds the scope of this paper. Note that a potential perpendicular surface anisotropy affects the thin film as well and does therefore not account for the additional magnetization reduction in the pillar.

Concerning the boundary conditions and exchange stiffness we have come to the following conclusions: Strong pinning can be ruled out in our pillars (see section VI); reasonable agreement between the experimental data and the model is obtained under the assumption of weak asymmetric pinning. The pinning parameter deduced from the extracted mode numbers is with $d \leq 1$ about 10 times smaller than the one calculated by means of eq. (5) in Ref. 9 ($d \approx 10$) when using the material parameters of Tab. II and zero or small perpendicular anisotropy. Any value of $d$ substantially larger than 3 is found to yield mode numbers for the lowest mode very close to 1, i.e. strong pinning. This discrepancy between our result and the predictions of Guslienko’s analytical model is not understood, since the latter is expected to be valid in the regime of element thicknesses smaller than the exchange length as well. We emphasize that, just as the magnetization, the exchange stiffness of the free layer does not exceed $2/3$ of the thin film value, independent of the boundary conditions. The magnetic properties of the nanopillar can therefore by no means be described by the values measured on the corresponding unprocessed thin films. The mutual dipolar coupling terms account for the following three features observed in the experimental spectra: The shift of the EA hysteresis loops (and consequently the EA spectra) is caused by a net dipolar field created by the SAF layers favoring the antiparallel configuration of the free layer and the reference layer magnetizations. The gaps in the mode $F_0$ in the EA spectra stem from the anticrossing of this mode with the acoustic SAF modes. Finally, in the HA spectra, the mutual dipolar coupling of the FL and the SAF raises the minima of $F_0$ in frequency and pushes them to higher fields. Within the diagonal tensor approximation of the dipolar coupling used in the model, the tensor components turned out to be uniquely determined by the experimental data. They are found to be significantly smaller than the values predicted by the formalism developed by Newell et al. or by the simplified version using for the in-plane components of the mutual dipolar coupling tensor the corresponding components of the self-demagnetizing tensor, as commonly practiced when modeling flip-flop switching in MRAM cells. E.g. the mutual dipolar coupling between the free layer and the reference layer of our pillars would be...
overestimated by Refs. 21 and 34,36 by a factor of 4 or 7, respectively. A possible explanation for this discrepancy may be that the mutual dipolar coupling field corresponding to the extracted tensor components in Tab. II is an averaged effective dipolar coupling field comprising the dipolar coupling due to the charges at the lateral layer boundaries as well as some Néel-type coupling resulting from the correlated roughness of the three magnetic layers. This orange-peel coupling may partially compensate the antiparallel coupling due to the charges at the lateral edges.

Finally, the extracted exchange bias energy and the interlayer exchange coupling are consistent with the large body of dedicated literature.

B. Low-field behavior and its relevance for applications

In section II we have seen that at low fields both EA and HA spectra show for all three pillar sizes unmistakable signs of nonuniform magnetizations: EA spectra contain edge modes in the low-field region of the P state accompanied by a reduced remanence; HA spectra possess at zero-field, instead of the local maximum predicted by the model, a sharp minimum. With increasing pillar size the modes on both EA and HA become progressively deformed around zero field, indicating increasing nonuniformity of the magnetization. (It is interesting to note that the minimum at zero HA field is not observed in the otherwise very similar HA spectrum of the three magnetic layers. This orange-peel coupling may partially compensate the antiparallel coupling due to the charges at the lateral edges.

Moreover, it has been demonstrated in section III C that for the largest pillar the frequency of the mode F0 at zero-field is history dependent, which we had interpreted with the existence of nearly degenerate micromagnetic configurations in the low field region. It is possible that at zero field, the Oersted field created by the bias current may after all affect the micromagnetic state of the layers, though it is negligible otherwise.

The nonuniformities of the magnetizations are expected to influence the switching dynamics of the pillar. The first consequence is that they lower the coercivity field, thus enlarging its difference to the shape anisotropy field. Indeed, this effect has been found to be particularly strong for size L (see section II), whereas it is negligible for size S. More importantly, the fact that the lowest mode is not the uniform mode, but in most cases an edge mode, will affect the magnetization reversal path in current-induced switching, favoring nonuniform reversal paths, as has already been concluded indirectly from reversal speed experiments. 37

C. Spin wave phenomena beyond the analytical approximations

Based on the assumption of macrospin equilibrium magnetizations, our model is certain not to describe any effect resulting from nonuniformities of the magnetization. This is the case e.g. for the low-field anomaly, discussed in the previous paragraph. As the nonuniformity of the magnetization increases with the pillar size, it starts to affect the medium and finally even the high field regions of the spectra. Consequently, for the larger pillar sizes, the calculated spectra show large deviations from their experimental counterparts in all field regions. E.g., on the HA, the saturation minima of the lowest FL modes are observed at much higher fields than predicted, while on the EA, the observed mode spacing is considerably larger than the maximum possible mode spacing predicted by the model (even in the unlikely case of total pinning). Further inconsistencies concerning the mode spacing become obvious in EA and HA spectra of the largest pillar, where in the experimental EA spectrum successive FL modes (F0 and F1) have a spacing of 6 GHz compared to 2 GHz and less for the SAF modes or for the FL modes in the HA spectra, while theory predicts approximately the same mode spacing for FL and SAF modes, on both the EA and the HA. Furthermore, the good visibility of the higher modes in the P state of the EA spectra on the one hand, and their complete invisibility in the low field AP state on the other hand, are in qualitative disagreement with our considerations in section V B. Finally, the systematic asymmetry of the HA spectra (discussed in detail at the end of section VI) cannot be explained within our model.

VIII. CONCLUSIONS

In this paper, we have studied the magnetic field dependence of the mode frequency of thermally excited spin waves in rectangular shaped MgO-MTJ nanopillars of different lateral sizes. The spin wave spectra (frequency versus easy and hard axis applied field) of individual devices were obtained using spectrally resolved electrical noise power measurements. In all spectra, several independent quantized spin wave modes stemming from eigenexcitations in the free layer and the SAF layers of the MTJ have been observed. By diagonalizing the dynamical matrix of a system of three coupled, spatially confined magnetic layers, we have modeled the mode frequencies for the smallest pillar size, \(60 \times 100 \text{ nm}^2\), obtaining quantitative agreement at high and medium applied fields. Our ability to detect a given spin wave mode depends on its character (i.e. spatial distribution) as well as the micromagnetic configuration of the layers. With the help of these discrimination criteria, we could identify the various modes and extract the material parameters of the pillar (Tab. II). The magnetizations and the exchange stiffness constants were found to be significantly reduced compared to the corresponding thin film values, while the interlayer exchange coupling and the exchange bias are essentially consistent with their thin film counterparts. The interlayer dipolar coupling between the different layers could be well described in terms of an effective mutual dipolar coupling of the different layers. Moreover, we could infer that the pinning of the magnetizations at the lateral boundaries must be weak and slightly asymmetric.

Finally, at low fields and for larger pillar sizes, there is clear evidence for strong nonuniformities of the layer magnetiza-
tions, leading to quantitative differences between the calculated and measured spin wave frequencies. In these cases, the calculation of the eigenmodes of the nanopillar requires micromagnetic simulations, which exceeds the scope of this study.

Acknowledgments

We thank Singulus Technologies A.G. for the layer deposition in a Timaris PVD system. A. H. is supported by the European Community (EC) under the 6th FP for the Marie Curie RTN SPINSWITCH, contract no. MRTN-CT-2006-035327. The work in Leuven was supported by the EC program IST STREP, under contract no. IST-016939 TUNAMOS; S. C. acknowledges IWT Flanders for financial support.

7 Electronic address: annerose.helmer@u-psud.fr
9 C. Chappert, A. Fert, and F. N. Van Dau, NATURE MATERIALS 6, 813 (2007), ISSN 1476-1122.